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USING THE ANALYTIC HIERARCHY PROCESS FOR DECISION MAKING IN ENGINEERING APPLICATIONS: SOME CHALLENGES

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In many industrial engineering applications the final decision is based on the evaluation of a number of alternatives in terms of a number of criteria. This problem may become a very difficult one when the criteria are expressed in different units or the pertinent data are difficult to be quantified. The Analytic Hierarchy Process (AHP) is an effective approach in dealing with this kind of decision problems. This paper examines some of the practical and computational issues involved when the AHP method is used in engineering applications.

Significance: In many engineering applications the final decision depends on the evaluation of a set of alternatives in terms of a number of decision criteria. This may be a difficult task and the Analytic Hierarchy Process seems to provide an effective way for properly quantifying the pertinent data. However, there are many critical issues that a decision maker needs to be aware of.

Key words: Multi-Criteria Decision-Making, Analytic Hierarchy Process, Pairwise Comparisons.

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1. INTRODUCTION

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach and was introduced by Saaty (1977 and 1994). The AHP has attracted the interest of many researchers mainly due to the nice mathematical properties of the method and the fact that the required input data are rather easy to obtain. The AHP is a decision support tool which can be used to solve complex decision problems. It uses a multi-level hierarchical structure of objectives, criteria, subcriteria, and alternatives. The pertinent data are derived by using a set of pairwise comparisons. These comparisons are used to obtain the weights of importance of the decision criteria, and the relative performance measures of the alternatives in terms of each individual decision criterion. If the comparisons are not perfectly consistent, then it provides a mechanism for improving consistency.

Some of the industrial engineering applications of the AHP include its use in integrated manufacturing (Putrus, 1990), in the evaluation of technology investment decisions (Boucher and McStravic, 1991), in flexible manufacturing systems (Wabalickis, 1988), layout design (Cambron and Evans, 1991), and also in other engineering problems (Wang and Raz, 1991).

As an illustrative application consider the case in which one wishes to upgrade the computer system of a computer integrated manufacturing (CIM) facility. There is a number of different configurations available to choose from. The different systems are the alternatives. A decision should also consider issues such as: cost, performance characteristics (i.e.,

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CPU speed, memory capacity, RAM, etc.), availability of software, maintenance, expendability, etc. These may be some of the decision criteria for this problem. In the above problem we are interested in determining the **best alternative** (i.e., computer system). In some other situations, however, one may be interested in determining the relative importance of all the alternatives under consideration. For instance, if one is interested in funding a set of competing projects (which now are the alternatives), then the relative importance of these projects is required (so the budget can be distributed proportionally to their relative importance).

Multi-criteria decision-making (MCDM) plays a critical role in many real life problems. It is not an exaggeration to argue that almost any local or federal government, industry, or business activity involves, in one way or the other, the evaluation of a set of alternatives in terms of a set of decision criteria. Very often these criteria are conflicting with each other. Even more often the pertinent data are very expensive to collect.

2. STRUCTURE OF THE DECISION PROBLEM UNDER CONSIDERATION

The structure of the typical decision problem considered in this paper consists of a number, say M, of alternatives and a number, say N, of decision criteria. Each alternative can be evaluated in terms of the decision criteria and the relative importance (or weight) of each criterion can be estimated as well. Let a_{ij} $(i = 1,2,3,\ldots,M,$ and $N = 1,2,3,\ldots,N)$ denote the **performance value** of the i-th alternative (i.e., A_i) in terms of the j-th criterion (i.e., C_j). Also denote as W_j the **weight** of the criterion C_j. Then, the core of the typical MCDM problem can be represented by the following **decision matrix**:

		Criterion			
	C_1	C_2	C_3	…	C_N
Alt.	W_1	W_2	W_3	…	W_N
A_1	a_{11}	a_{12}	a_{13}	…	a_{1N}
A_2	a_{21}	a_{22}	a_{23}	…	a_{2N}
A_3	a_{31}	a_{32}	a_{33}	…	a_{3N}
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
A_M	a_{M1}	a_{M2}	a_{M3}	…	a_{MN}

Given the above decision matrix, the decision problem considered in this study is how to determine which is the best alternative. A slightly different problem is to determine the relative significance of the M alternatives when they are examined in terms of the N decision criteria combined.

In a simple MCDM situation, all the criteria are expressed in terms of the same unit (e.g., dollars). However, in many real life MCDM problems different criteria may be expressed in different dimensions. Examples of such dimensions include dollar figures, weight, time, political impact, environmental impact, etc. It is this issue of multiple dimensions which makes the typical MCDM problem to be a complex one and the AHP, or its variants, may offer a great assistance in solving this type of problems.

3. THE ANALYTIC HIERARCHY PROCESS

The AHP and its use of pairwise comparisons has inspired the creation of many other decision-making methods. Besides its wide acceptance, it also created some considerable criticism; both for theoretical and for practical reasons. Since the early days it became apparent that there are some problems with the way pairwise comparisons are used and the way the AHP evaluates alternatives. First, Belton and Gear (1983) observed that the AHP may reverse the ranking of the alternatives when an alternative identical to one of the already existing alternatives is introduced. In order to overcome this deficiency, Belton and Gear proposed that each column of the AHP decision matrix to be divided by the maximum entry of that column. Thus, they introduced a variant of the original AHP, called the **revised-AHP**. Later, Saaty (1994) accepted the previous variant of the AHP and now it is called the **Ideal Mode AHP**. Besides the revised-AHP, other authors also introduced other variants of the original AHP. However, the AHP (in the original or in the ideal mode) is the most widely accepted method and is considered by many as the most reliable MCDM method.

The fact that rank reversal also occurs in the AHP when near copies are considered, has also been studied by Dyer and Wendell (1985). Saaty (1983a and 1987) provided some axioms and guidelines on how close a near copy can be to an original alternative without causing a rank reversal. He suggested that the decision maker has to eliminate alternatives from consideration that score within 10 percent of another alternative. This recommendation was later sharply criticized by Dyer

(1990). The first step in the AHP is the estimation of the pertinent data. That is, the estimation of the a_{ij} and W_j values of the decision matrix. This is described in the next sub-section.

Table 1: Scale of Relative Importances (according to Saaty (1980))

Intensity of Importance	**Definition**	**Explanation**
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between the two adjacent judgments	When compromise is needed
Reciprocals of above nonzero	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.	

3.1. The Use of Pairwise Comparisons

One of the most crucial steps in many decision-making methods is the accurate estimation of the pertinent data. This is a problem not bound in the AHP method only, but it is crucial in many other methods which need to elicit qualitative information from the decision-maker. Very often qualitative data cannot be known in terms of absolute values. For instance, *"what is the worth of a specific computer software in terms of a user adaptivity criterion?"* Although information about questions like the previous one are vital in making the correct decision, it is very difficult, if not impossible, to quantify them correctly. Therefore, many decision-making methods attempt to determine the **relative** importance, or weight, of the alternatives in terms of each criterion involved in a given decision-making problem.

An approach based on pairwise comparisons which was proposed by Saaty (1980) has long attracted the interest of many researchers. Pairwise comparisons are used to determine the relative importance of each alternative in terms of each criterion. In this approach the decision-maker has to express his opinion about the value of one single pairwise comparison at a time. Usually, the decision-maker has to choose his answer among 10-17 discrete choices. Each choice is a linguistic phrase. Some examples of such linguistic phrases are: *"A is more important than B"*, or *"A is of the same importance as B"*, or *"A is a little more important than B"*, and so on (see also table 1).

The main problem with the pairwise comparisons is **how to quantify** the linguistic choices selected by the decision maker during their evaluation. All the methods which use the pairwise comparisons approach eventually express the qualitative answers of a decision maker into some numbers which, most of the time, are ratios of integers. A case in which pairwise comparisons are expressed as differences (instead of ratios) was used to define similarity relations and is described by Triantaphyllou (1993). The following paragraphs examine the issue of quantifying pairwise comparisons. Since pairwise comparisons are the keystone of these decision-making processes, correctly quantifying them is the most crucial step in multi-criteria decision-making methods which use qualitative data.

Pairwise comparisons are quantified by using a **scale**. Such a scale is an one-to-one mapping between the set of discrete linguistic choices available to the decision maker and a discrete set of numbers which represent the importance, or weight, of the previous linguistic choices. The scale proposed by Saaty is depicted in table 1. Other scales have also been proposed by others. An evaluation of 78 different scales appears in Triantaphyllou et al. (1994). All the alternative scales depart from some psychological theories and develop the numbers to be used based on these psychological theories.

In 1846 Weber stated his law regarding a stimulus of measurable magnitude. According to his law a change in sensation is noticed if the stimulus is increased by a constant percentage of the stimulus itself (Saaty, 1980). That is, people are unable to make choices from an infinite set. For example, people cannot distinguish between two very close values of importance, say 3.00 and 3.02. Psychological experiments have also shown that individuals cannot simultaneously compare more than seven objects (plus or minus two) (Miller, 1956). This is the main reasoning used by Saaty to establish 9 as the upper limit of his scale, 1 as the lower limit and a unit difference between successive scale values.

The values of the pairwise comparisons in the AHP are determined according to the scale introduced by Saaty (1980). According to this scale, the available values for the pairwise comparisons are members of the set: {9, 8, 7, 6, 5, 4, 3, 2, 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9} (see also table 1).

As an illustrative example consider the following situation. Suppose that in the previous example of selecting the best computer system, there are three alternative configurations, say A, B, and C. Also, suppose that one of the decision criteria is hardware expandability (i.e., the flexibility of attaching to the system other related peripheral devices, such as printers, new memory, etc.). Suppose that system A is much better than system B, and system C is the least desired one as far as the hardware expandability criterion is concerned. Suppose that following is the judgement matrix when the three alternative configurations are examined in terms of this criterion.

C_1: Hardware Expandability	**A**	**B**	**C**
A	1	6	8
B	1/6	1	4
C	1/8	1/4	1

For instance, when system A is compared to system B then the decision-maker has determined that system A is between to be classified as *"essentially more important"* and *"demonstrated more important"* than system B (see also table 1). Thus, the corresponding comparison assumes the value of 6. A similar interpretation is true for the rest of the entries.

The next step is to extract the relative importances implied by the previous comparisons. That is, how important are the three alternatives when they are considered in terms of the hardware expandability criterion? Saaty asserts that to answer this question one has to estimate the **right principal eigenvector** of the previous matrix. Given a judgment matrix with pairwise comparisons, the corresponding maximum left eigenvector is approximated by using the geometric mean of each row. That is, the elements in each row are multiplied with each other and then the n-th root is taken (where n is the number of elements in the row). Next the numbers are normalized by dividing them with their sum. Hence, for the previous matrix the corresponding priority vector is: (0.754, 0.181, 0.065).

An evaluation of the eigenvalue approach can be found in (Triantaphyllou and Mann, 1990). An alternative approach for evaluating the relative priorities from a judgment matrix is based on a least squares formulation and is described in (Triantaphyllou et al., 1990a and 1990b). One of the most practical issues in the AHP methodology is that it allows for slightly non-consistent pairwise comparisons. If all the comparisons are perfectly consistent, then the following relation should always be true for any combination of comparisons taken from the judgment matrix: $a_{ij} = a_{ik} a_{kj}$.

However, perfect consistency rarely occurs in practice. In the AHP the pairwise comparisons in a judgment matrix are considered to be adequately consistent if the corresponding consistency ratio (CR) is less than 10% (Saaty, 1980). The CR coefficient is calculated as follows. First the consistency index (CI) needs to be estimated. This is done by adding the columns in the judgment matrix and multiply the resulting vector by the vector of priorities (i.e., the approximated eigenvector) obtained earlier. This yields an approximation of the maximum eigenvalue, denoted by λ_{max}. Then, the CI value is calculated by using the formula: CI $= (\lambda_{max} - n)/(n - 1)$. Next the consistency ratio CR is obtained by dividing the

CI value by the Random Consistency index (RCI) as given in table 2.

When these approximations are applied to the previous judgment matrix it can be verified that the following are derived: λ_{max} = 3.136, CI = 0.068, and CR = 0.117. If the CR value is greater than 0.10, then it is a good idea to study the problem further and re-evaluate the pairwise comparisons (this was not done in the numerical example in this paper).

Table 2: RCI values for different values of *n*.

n	1	2	3	4	5	6	7	8	9
RCI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

After the alternatives are compared with each other in terms of each one of the decision criteria and the individual priority vectors are derived, the synthesis step is taken. The priority vectors become the columns of the decision matrix (not to be confused with the judgment matrices with the pairwise comparisons). The weights of importance of the criteria are also determined by using pairwise comparisons. Therefore, if a problem has M alternatives and N criteria, then the decision maker is required to construct N judgment matrices (one for each criterion) of order MxM and one judgment matrix of order NxN (for the N criteria). Finally, given a decision matrix the final priorities, denoted by A^{i}_{AHP}, of the alternatives in terms of all the criteria combined are determined according to the following formula (1).

$$A^{i}_{AHP} = \sum_{j=1}^{N} a_{ij}\, w_j, \quad \text{for } i = 1, 2, ,3, ,... M. \quad \ldots \tag{1}$$

Recall that in the case of the **ideal mode** AHP the columns of the decision matrix are normalized by dividing by the largest entry in each column. All the above are further illustrated in terms of a numerical example. The numerical data for this example were adapted from an example given in (Saaty, 1983b).

3.2. A Numerical Example

Suppose that the three alternative computer systems described earlier need to be evaluated in terms of the four decision criteria: hardware expandability, hardware maintainability, financing available, and user friendly characteristics of the operating system and related available software. If more criteria are required to be considered, then this example can be expanded accordingly. Suppose that the following matrices represent the corresponding judgment matrices with the pairwise comparisons. Note that the corresponding priority vectors (for the individual criteria) and the consistency coefficients are given as well. The first such matrix is the same as the one analyzed in the previous sub-section and therefore it is omitted in this section (recall that the priority vectors is: (0.754 0.181 0.065) and λ_{max} = 3.136, CI = 0.068, and CR = 0.117).

C_2: Hardware Maintainability	**A**	**B**	**C**	**Priority Vector**
A	1	7	1/5	0.233
B	1/7	1	1/8	0.055
C	5	8	1	0.713

λ_{max} = 3.247, CI = 0.124, and CR = 0.213.

C_3: Financing Available	**A**	**B**	**C**	**Priority Vector**
A	1	8	6	0.745
B	1/8	1	1/4	0.065
C	1/6	4	1	0.181

λ_{max} = 3.130, CI = 0.068, and CR = 0.117.

C_4: User Friendly	**A**	**B**	**C**	**Priority Vector**
A	1	5	4	0.674
B	1/5	1	1/3	0.101
C	1/4	3	1	0.226

λ_{max} = 3.086, CI = 0.043, and CR = 0.074.

Finally, the following is the judgment matrix for the case of comparing the importances of the four decision criteria.

The four Criteria	**C_1**	**C_2**	**C_3**	**C_4**	**Priority Vector**
C_1	1	5	3	7	0.553
C_2	1/5	1	1/3	5	0.131
C_3	1/3	3	1	6	0.271
C_4	1/7	1/5	1/6	1	0.045

λ_{max} = 4.252, CI = 0.084, and CR = 0.093.

As it was mentioned earlier, the previous priority vectors are used to form the entries of the decision matrix for this problem. The decision matrix and the resulted final priorities (which are calculated according to formula (1)) are as follows:

Decision Matrix and Solution when the Original AHP is used:

	Criterion				
	C_1	**C_2**	**C_3**	**C_4**	
Alt.	(0.553	0.131	0.271	0.045)	**Final Priority**
A	0.754	0.233	0.745	0.674	0.680
B	0.181	0.055	0.065	0.101	0.130
C	0.065	0.713	0.181	0.226	0.190

Decision Matrix and Solution when the Ideal Mode AHP is used:

	Criterion					
	C_1	**C_2**	**C_3**	**C_4**		
Alt.	(0.553	0.131	0.271	0.045)	**Final Priority**	**After Normalization**
A	1.000	0.327	1.000	1.000	0.912	0.678
B	0.240	0.077	0.087	0.150	0.173	0.129
C	0.086	1.000	0.243	0.335	0.260	0.193

Therefore, the best system is A followed by system C which is followed by system B. Observe that although both the original AHP and the ideal mode AHP yielded the same ranking for the three alternatives, they assigned different final priorities for these alternatives. The next section describes some cases in which the original AHP (and in a similar manner the ideal mode AHP) can exhibit ranking abnormalities for some decision making problems.

4. SOME CASES OF NUMERICAL INSTABILITY WITH THE AHP

In Triantaphyllou and Mann (1989) the four multi-criteria decision-making methods: the AHP, revised AHP (i.e., the ideal mode AHP), the Weighted Sum Model (WSM) (Fishburn, 1967), and the Weighted Product Model (WPM) (Miller and Starr, 1969) were examined in terms of two evaluative criteria. That study focused on the last step of any MCDM method which involves the processing of the final decision matrix. That is, given the weights of relative performance of the decision criteria, and the performance of the alternatives in terms of each one of the decision criteria, then determine what is the ranking (or relative priorities) of the alternatives.

As it was shown in Triantaphyllou and Mann (1989), however, these methods can give different answers to the same problem. Since the truly best alternative is the same **regardless** of the method chosen, an estimation of the accuracy of each method is highly desirable. The most difficult problem that arises here is how one can evaluate a multi-dimensional decision-making method when the true best alternative is not known. Two evaluative criteria were introduced for the above purpose.

The **first evaluative criterion** has to do with the premise that ***a method which is accurate in multi-dimensional problems should also be accurate in single-dimensional problems. There is no reason for an accurate multi-dimensional method to fail in giving accurate results in single-dimensional problems, since single-dimensional problems are special cases of multi-dimensional ones.*** Because the first method, the WSM, gives the most acceptable results for the majority of single-dimensional problems, the result of the WSM was used as the standard for evaluating the other three methods in this context.

The **second evaluative criterion** considers the premise that ***a desirable method should not change the indication of the best alternative when an alternative (not the best) is replaced by another worse alternative (given that the importance of each criterion remains unchanged)***.

In the following paragraphs we briefly present some numerical cases in which the original AHP fails when it is examined in terms of the previous two evaluative criteria. The case of the ideal mode AHP is similar (although different numerical examples are needed to expose cases of numerical instability), and thus it is omitted for simplicity.

4.1 Testing the original AHP Using the First Evaluative Criterion

Example 1: Suppose that the matrix below depicts the actual values, **measured in the same units** (for instance, in US dollars) of three alternatives A_1, A_2, and A_3, in terms of three criteria with the following weights of importance: $w_1 = 8/13$, $w_2 = 2/13$, and $w_3 = 3/13$. Suppose that these criteria are benefit criteria. That is, the higher the value the better it is. Problems like this one are very common when one wishes, for instance, to perform an engineering economy study.

	Criteria		
	C_1	C_2	C_3
Alter.	(8/13	2/13	3/13)
A_1	1	9	9
A_2	5	2	2
A_3	1	5	9

Given the above data it is easy to see that the final scores of the alternatives in terms of the three criteria are 53/13, 50/13, and 45/13, respectively. For instance, the score for the first alternative is: $1(8/13) + 9(2/13) + 9(3/13) = 53/13$. Therefore, alternative A_1 is the best one (since it corresponds to the score 53/13). The decision method used above is the weighted sum model (WSM) which is the most commonly used approach when all the units are the same (as it was assumed in this example).

When the AHP approach is used, the previous data have to be normalized. If the original AHP is to be used, then the above data are normalized by dividing each column by the sum of the elements in that column. If the data cannot be obtained directly, then the method which is based on the pairwise comparisons needs to be employed. Since there are three criteria, the decision maker needs to construct three matrices with pairwise comparisons of size 3x3 each. The three 3x3 matrices with the pairwise comparisons that correspond to this problem are as follows (perfect consistency in the pairwise comparisons is assumed in order to block out any effects due to inconsistent comparisons):

Criterion C_1 **Criterion C_2** **Criterion C_3**

$$\begin{bmatrix} 1 & 1/5 & 1/1 \\ 5/1 & 1 & 5/1 \\ 1/1 & 1/5 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 9/2 & 9/5 \\ 2/9 & 1 & 2/5 \\ 5/9 & 5/2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 9/2 & 9/9 \\ 2/9 & 1 & 2/9 \\ 9/9 & 9/2 & 1 \end{bmatrix}$$

Therefore, the MxN (i.e., 3x3) matrix with the relative (i.e., normalized) importances of the alternatives in terms of each criterion that is used in the final step of the AHP is:

	Criteria		
C_1	C_2	C_3	
Alter.(	8/13	2/13	3/13)
A_1	1/7	9/16	9/20
A_2	5/7	2/16	2/20
A_3	1/7	5/16	9/20

Applying the last step of the AHP it turns out that the alternative A_2 is the best one ($A^2_{AHP} = A^*_{AHP} = 0.48$). Obviously, this is in **<u>contradiction</u>** with the conclusion derived earlier by using the WSM. ■

4.2 Testing the original AHP Using the Second Evaluative Criterion

Example 2: Suppose that the following is a matrix of the eigenvectors produced by the AHP process. That is, the matrix contains relative values for the importance of the alternatives instead of the actual values. Assume that the criteria have weights $w_1 = 2/7$, $w_2 = 2/7$, and $w_3 = 3/7$.

	Criteria			
	C_1	C_2	C_3	
Alter.(	2/7	2/7	3/7)	
A_1	9/19	2/12	2/7	
A_2	5/19	1/12	4/7	= Matrix M1
A_3	5/19	9/12	1/7	

It can be shown (by multiplying the matrix with the relative importances by the vector with the weights of the 3 criteria followed by normalization) that the priority vector of the alternatives (according to the original AHP) is (0.305, 0.344, 0.351).

Apparently, the best alternative is A_3. If in the above problem the alternative A_1 (which is not the best one and was defined by the relative values (9/19 2/12 2/7), is replaced by A'_1 which is worse than the original alternative A_1, then, the above matrix is modified as follows:

	Criteria			
	C_1	C_2	C_3	
Alter.(	2/7	2/7	3/7)	
A'_1	8/18	1/11	1/6	
A_2	5/18	1/11	4/6	= Matrix M2
A_3	5/18	9/11	1/6	

Matrix M1 can be considered as the matrix with the relative values obtained from the following three 3x3 matrices with pairwise comparisons (perfect consistency in the pairwise comparisons is again assumed as before):

Criterion C_1 **Criterion C_2** **Criterion C_3**

$$\begin{bmatrix} 1 & 9/5 & 9/5 \\ 5/9 & 1 & 5/5 \\ 5/9 & 5/5 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2/1 & 2/9 \\ 1/2 & 1 & 1/9 \\ 9/2 & 9/1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2/4 & 2/1 \\ 4/2 & 1 & 4/1 \\ 1/2 & 1/4 & 1 \end{bmatrix}$$

Matrix M2 has been derived from matrix M1 by substituting the alternative A_1 with the lesser A'_1 = (8/18 1/11 1/6) < (9/19 2/12 2/7). That is, instead of 9 it is now 8, instead of 2 it is now 1, and instead of 19 it is now 18.

Similarly, the priority vector for matrix M2 is (0.224, 0.391, 0.385). It is clear that now the best alternative is A_2. The last statement is, obviously, in **contradiction** with the original result namely, that the best alternative is A_3. Thus, by introducing a new worse alternative (different from the best one) it is possible to have a change in the indication of the best alternative. ■

In (Triantaphyllou and Mann, 1989) the previous two evaluative criteria were applied on random test problems with the numbers of decision criteria and alternatives taking the values 3, 5, 7, ..., 21. In those experiments it was found that all the previous four MCDM methods (i.e., the AHP; both original and in ideal mode, the WSM and the WPM) were inaccurate. Furthermore, these results were used to form a decision problem in which the four methods themselves were the alternatives. The decision criteria were derived by considering the previous two evaluative criteria.

Since the best decision making method was not known, and each alternative was a decision making method, we solved the previous problem by using each one of the rival methods. To our greatest surprise, one method would recommend another, rival method, as being the best method! This was reported in (Triantaphyllou and Mann, 1989) as a **decision making paradox**. However, the final results seemed to suggest that the **revised AHP** (i.e., the ideal mode AHP) was the most efficient MCDM method of the ones examined. Finally, a different approach of evaluating the performance of the original AHP and the ideal mode AHP, by using a continuity assumption, is described by Triantaphyllou and Mann (1994a). In that treatment it was found that these two methods may yield dramatically inaccurate results (more than 80% of the time).

5. CONCLUSIONS AND DISCUSSION

The AHP provides a convenient approach for solving complex MCDM problems in engineering. It should be noted that there is a software package, called ***Expert Choice*** (1990), which has significantly contributed to the wide acceptance of the AHP methodology. However, as this paper demonstrated with some illustrative examples, its use to engineering problems should be a cautious one. There is sufficient evidence to suggest that the recommendations made the AHP should not be taken literally. In matter of fact, the closer the final priority values are with each other, the more careful the user should be.

This is true with any MCDM method. The numerical examples in this paper, along with the extensive research of the authors in this area (please also see the reference list for more details), strongly suggest that when some alternatives appear to be very close with each other, then the decision-maker needs to be very cautious. An apparent remedy is to try to consider additional decision criteria which, hopefully, can assist in drastically discriminating among the alternatives. A summary of the results of a number of studies on the AHP and pairwise comparisons by the authors can be found in (Triantaphyllou and Mann, 1994b).

The above observations suggest that MCDM methods should be used as **decision support tools** and not as the means for deriving the final answer. To find the truly best solution to a MCDM problem may never be humanly possible. The conclusions of the solution should be taken lightly and used only as indications to what may be the best answer. Although the search for finding the best MCDM method may never end, research in this area of decision-making is still critical and very valuable in many scientific and engineering applications.

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